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WHAT'S THE GAME TEAM OWNERS PLAY?

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SUMMARY: 1. Introduction – 2. The Contest Success Function (CSF) – 3. The Win Percent Function (WPF) – 4. Conclusion – References

JEL CLASSIFICATION: L83, D7

1. *Introduction*

In the recent literature of professional team sports, a discussion has emerged among sports economists regarding the most appropriate approach to analyse the player labour market in team sports. Should it be, or can it be, the well-known Walrasian competitive equilibrium model or should it be a Nash equilibrium model, that is: the outcome of a game where each team's strategy is affected by the strategy of the other teams.[1] It all started with the criticism of the Walrasian model by Szymanski

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[1] See on this point S. SZYMANSKI, *The Economic Design of a Sporting Contests*, *J. of Ec. Lit.*, vol. 41, n. 4, 2003, 1137-87; S. SZYMANSKI, *Professional Team Sports are a Only Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium and the Invariance Principle*, *J. of Sp. Ec.*, vol. 5, n. 2, 2004, 111-26; S. SZYMANSKI, S. KESENNE, *Competitive Balance and Gate Revenue Sharing in Team Sports*, *J. of Ind. Ec.*, vol. 51, n. 4, 2004, 513-25; W. ECKARD, Comment: "*Professional Team Sports Are Only a Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium and the Invariance Principle", J. of Sp. Ec.*, vol. 7, n. 2, 2006, 234-239; R. FORT, *Talent Market Models in North American and World Leagues*, in P. Rodriguez, S. Kesenne, J. Garcia (eds;), *Sports Economics after Fifty Years: Essays in Honour of Simon Rottenberg*, Oviedo University Press, Oviedo, 2007, 83-106; R. FORT, J. QUIRK, *Rational Expectations and Pro Sports Leagues*, *Scot. J. of Pol. Ec.*, vol. 54, 2007, 377-390; S. KESENNE, *The Economic Theory of Professional Team Sports, an analytical treatment*, Edward Elgar Publ., Cheltenham, 2007; R. FORT, J. WINFREE, *Sports Really are Different: The Contest Success Function,*

and Kesenne in the Journal of Industrial Economics in 2004, arguing that the conventional textbook approach of a competitive equilibrium model is less appropriate (Kesenne) or totally inappropriate (Szymanski) to describe and explain what is happening in that market. The authors assert that a non-cooperative Nash equilibrium model is the appropriate approach to analyse a sports league. The distinction is not unimportant because, for these two models, different results have been derived with respect to the distribution of talent, the average salary level and the impact of revenue sharing. In this short contribution, we compare the contest success function with the correct win percent function and try to show that, with the correct relationship between wining percentage and talent, it is always a game.

2. *The Contest Success Function (CSF)*

All sports economists seem to agree that the Nash model is the right model if the supply of talent is elastic, which is the case in the open European player market after the Bosman verdict in 1995. If the supply of talent is inelastic, as in the North American major leagues, the advocates of the two models disagree. The American sports economists seem to have reached the conclusion that the Walrasian model and the Nash model are just two different models. Szymanski,[2] however, rejects the Walrasian model, also if the supply of talent is constant. He argues that in this model n-1 teams can simply choose their own winning percentage, which is somewhat odd indeed, and, moreover, that the n^{th} team has no choice of strategy because the winning percentages have to add up to a constant which is *n/2*.

Two different assumptions can be made regarding the clubs' conjectures when hiring talent.

One assumption is the so-called Nash conjecture, meaning that for all teams $i \neq j \;\; \frac{\partial t_j}{\partial t_i} = 0$ where *t* indicates the number of playing talents. This certainly applies if the talent supply is elastic. If one team hires a talent, it is obvious that another team does not have to lose a talent.

The other assumption is the so-called constant-supply conjecture: in the symmetric case, it simply means that for all teams $i \neq j \;\; \frac{\partial t_j}{\partial t_i} = -1/(n-1)$. In the non-symmetric case, it is likely that the talent stock of all other teams changes as $\frac{\partial t_j}{\partial t_i} = -t_j / (\frac{n}{2} - t_i)$, but it is also possible that one opponent team loses one talent

Marginal Product, and Marginal Revenue in *Pro Sports Leagues*, University of Michigan, Program in Sport Management, working paper, 2007.

[2] S. Szymanski, *Professional Team Sports are a Only Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium and the Invariance Principle*, cit.

while the talent stock of all other opponents does not change. As will be seen below, these alternative fixed-supply conjectures do not change the result. So, if the supply of talent is fixed, hiring talent implies that one or more teams are losing talent. The crucial question then seems to be: do club owners take into account, when calculating their demand for talent, that other teams lose talent. In other words, do teams internalize the externality caused by the hiring of talent?

If *w* stands for the winning percentage of a team, the conventional approach is to start from the contest success function (CSF), in its simple, or more general form with a power parameter that is different from one

$$w_i = \frac{n}{2}\frac{t_i}{\sum_{j=1}^{n} t_j} \quad \text{or} \quad w_i = \frac{n}{2}\frac{t_i^{\gamma}}{\sum_{j=1}^{n} t_j^{\gamma}} \quad \text{for all } i\text{: } 1, n \tag{1}$$

Because the winning percentages have to add up to *n/2*, the CSF is multiplied by this constant. One should realise, however, that is not a very good approximation; if one team has more than *2/n* % of the league's talents, its winning percentage can be larger than one. With the general form, all kinds of complications can show up, as shown by Fort and Winfree.[3] Continuing with the simple CSF, the marginal product of talent can be found as:

$$\frac{\partial w_i}{\partial t_i} = \frac{n}{2}\frac{\sum_{j=1}^{n} t_j - t_i\left(1+\sum_{j\neq i}^{n}\frac{\partial t_j}{\partial t_i}\right)}{\left(\sum_{j=1}^{n} t_j\right)^2} \tag{2}$$

With the constant-supply conjectural variation, $\sum_{j\neq i}^{n}\frac{\partial t_j}{\partial t_i} = -1$, one can find that (2) becomes:

$$\frac{\partial w_i}{\partial t_i} = \frac{n}{2}\frac{1}{\sum_{j=1}^{n} t_j} = 1 \tag{3}$$

with a constant talent supply equal to *n/2*. The implication is that one unit of talent increases the winning percentage by one unit, and that the winning percentage in a team's revenue function can be replaced by the number of talents. In other words, each owner can choose his team's winning percentage; it does not depend on the other teams' talents. So, the demand curves for talent of each team can be drawn as

[3] R. Fort, J. Winfree, *Sports Really are Different: The Contest Success Function, Marginal Product, and Marginal Revenue,* cit.

a function of the unit cost of talent. The market demand for talent is then simply the sum of the individual demand curves, which, facing the constant market supply of talent determines the market clearing cost of talent. This is the well-known Walrasian competitive equilibrium model. The owners take all the available information into account, including the implications of a fixed supply of talent. Under these perfectly competitive conditions, a Pareto-efficient point is reached, which guarantees the highest possible total league revenue. All teams' marginal revenues of winning are equal. Any deviation from this distribution of talents (or winning percentages) will result in a lower total league revenue. In our opinion, there is nothing fundamentally wrong with this model.

Under the Nash conjectural variation: $\sum_{j\neq i}^{n}\frac{\partial t_j}{\partial t_i}=0$, (2) becomes:

$$\frac{\partial w_i}{\partial t_i}=\frac{n}{2}\frac{\sum_{j\neq i}^{n}t_j}{(\sum_{j=1}^{n}t_j)^2} \tag{4}$$

Now, on first sight, the talent demand of each team is affected by the demand of the other teams in the league. So, it is a game and a non-cooperative Nash equilibrium model should be applied. However, with a constant supply of talent equal to *n/2*, expression (4) can be simplified to:

$$\frac{\partial w_i}{\partial t_i}=\frac{n-2t_i}{n} \tag{5}$$

The marginal product of talent is no longer a constant, the more talents a team has, the smaller the marginal product becomes, but, more importantly, the demand for talent is not affected by the talents of other teams. So, the argument goes, the Nash equilibrium model proposed by Szymanski and Kesenne[4] is inconsistent if the talent supply is constant. In that case, it is no longer a game. We have a Walrasian equilibrium, but now the team owners do not take into account that, with a constant talent supply, one more talent in their team means one less in another team. Not all available information is used and we do not find an efficient allocation of talent. There is a welfare loss in terms of total league revenue. In this latter market equilibrium, the distribution of talent is more equal and the market unit cost of talent is lower.[5] Moreover, revenue sharing will worsen the competitive

[4] S. Szymanski, S. Kesenne, *Competitive Balance and Gate Revenue Sharing in Team Sports, Journal of Industrial Economics*, cit.

[5] See S. Szymanski, *Professional Team Sports are a Only Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium and the Invariance Principle*, cit.; S. Kesenne,

balance under condition (4), whereas the invariance proposition holds under condition (3).

One can also argue that, if the number of teams in a league is large enough, the two equilibria coincide. As can be seen from expression (4) or (5): $if \ n \to \infty \ \ then \ \frac{\partial w_i}{\partial t_i} \to 1$.

So, if one starts from the CSF, a fixed-supply model is not a game, whether or not the constant supply is internalised.

3. *The Win Percent Function (WPF)*

The main weakness of the analysis above is that the CSF does not describe the correct relationship between a team's number of talents and the expected winning percentage. If we start from the correct relationship, it can be shown that it is always a game, whether or not the talent supply is constant. This relationship is:[6]

$$w_i = \frac{1}{n-1} \sum_{j \neq i}^{n} \frac{t_i}{t_i + t_j} \qquad \text{with} \quad \sum_{i=1}^{n} w_i = \frac{n}{2} \tag{6}$$

Furthermore, the talent ratio is no longer equal to the ratio of the winning percentages, or $t_i / t_j \neq w_i / w_j$, which is also more realistic. The marginal product of talent becomes now:

$$\frac{\partial w_i}{\partial t_i} = \frac{1}{n-1} \sum_{j \neq i}^{n} \frac{t_j - t_i \frac{\partial t_j}{\partial t_i}}{(t_i + t_j)^2} \tag{7}$$

With the Nash conjectural variation $\frac{\partial t_j}{\partial t_i} = 0$, we find that:

$$\frac{\partial w_i}{\partial t_i} = \frac{1}{n-1} \sum_{j \neq i}^{n} \frac{t_j}{(t_i + t_j)^2} \tag{8}$$

Given the (symmetric) fixed-supply conjectural variation $\frac{\partial t_j}{\partial t_i} = \frac{-1}{n-1}$, the marginal product of talent becomes:

The Economic Theory of Professional Team Sports, an analytical treatment. cit.

[6] L. Borghans, L. Groot, *The Competitive Balance Based on Team Quality*, Utrecht University, working paper, 2005.

$$\frac{\partial w_i}{\partial t_i} = \frac{1}{n-1}\sum_{j\neq i}^{n}\frac{t_j + \dfrac{t_i}{n-1}}{(t_i + t_j)^2} \tag{9}$$

Even with a constant talent supply, the marginal product of talent is still a function of the talents of the other teams. So, in both cases a game theoretic approach is more appropriate. A question is why this WPF is not used in all team sports models. The main reason probably is that, with this relationship, the analysis becomes too cumbersome mathematically.

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This analysis also shows that, with a simplified 2-club model, which is the approach used by many authors, the CSF and WPF coincide, whereas they are fundamentally different for an n-club model. In a 2-club model with a constant supply of talent, the marginal products of talent in (8) and (9) are no longer a function of the other team's talent. Even with the Nash conjectural variation, we can see that (8) reduces to $\dfrac{\partial w_i}{\partial t_i} = 1 - t_i$.

So, it is no longer a game. In both cases, a 2-club model turns out to be Walrasian. However, to the best of our knowledge, there does not exist a 2-club league in this world. What this analysis shows again is that starting form a 2-club model can be misleading, because all results derived from a 2-club model do not necessarily apply to an n-club model.

4. Conclusion

In the realistic case of an *n*-club league, using the correct relationship between a team's number of talents talent and its season winning percentage, it is clear that the only appropriate approach to analyse the player labour market is a the non-cooperative Nash equilibrium, whether or not the supply of playing talent is constant. The simplified 2-club model can present misleading results in this respect. Nevertheless, if one considers any other measure of the playing performance of a team, other than the season winning percentage, such as the CSF, in its simple or more general form, or some other indicator if playing performance, the Walrasian model can still be an appropriate approach if the supply of talent is constant.

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